RISK ASSESSMENT AND DECISION ANALYSIS WITH BAYESIAN NETWORKS Second Edition

RISK ASSESSMENT AND DECISION DECISION ANALYSIS WITH BAYESIAN DETWORKS

Second Edition

NORMAN FENTON MARTIN NEIL



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Foreword

Probabilistic models based on directed acyclic graphs have a long and rich tradition, beginning with work by the geneticist Sewall Wright in the 1920s. Variants have appeared in many fields. Within statistics, such models are known as directed graphical models; within cognitive science and artificial intelligence, such models are known as Bayesian networks. The name honors the Reverend Thomas Bayes (1702–1761), whose rule for updating probabilities in the light of new evidence is the foundation of the approach. The initial development of Bayesian networks in the late 1970s was motivated by the need to model the top-down (semantic) and bottom-up (perceptual) combination of evidence in reading. The capability for bidirectional inferences, combined with a rigorous probabilistic foundation, led to the rapid emergence of Bayesian networks as the method of choice for uncertain reasoning in AI and expert systems, replacing earlier adhoc rule-based schemes. Perhaps the most important aspect of Bayesian networks is that they are direct representations of the world, not of reasoning processes. The arrows in the diagrams represent real causal connections and not the flow of information during reasoning (as in rule-based systems or neural networks). Reasoning processes can operate on Bayesian networks by propagating information in any direction. For example, if the sprinkler is on, then the pavement is probably wet (prediction); if someone slips on the pavement, that also provides evidence that it is wet (abduction, or reasoning to a probable cause). On the other hand, if we see that the pavement is wet, that makes it more likely that the sprinkler is on or that it is raining (abduction); but if we then observe that the sprinkler is on, that reduces the likelihood that it is raining. It is the ability to perform this last form of reasoning—called explaining away—that makes Bayesian networks so powerful compared to rule-based systems or neural networks. They are especially useful and important for risk assessment and decision-making.

Although Bayesian networks are now used widely in many disciplines, those responsible for developing (as opposed to using) Bayesian network models typically require highly specialist knowledge of mathematics, probability, statistics, and computing. Part of the reason for this is that, although there have been several excellent books dedicated to Bayesian Networks and related methods, these books tend to be aimed at readers who already have a high level of mathematical sophistication—typically they are books that would be used at graduate or advanced undergraduate level in mathematics, statistics, or computer science. As such they are not accessible to readers who are not already proficient in those subjects. This book is an exciting development because it addresses this problem. While I am sure it would be suitable for undergraduate courses on probability and risk, it should be understandable by any numerate reader interested in risk assessment and decision making. The book provides sufficient motivation and examples (as well as the mathematics and probability where needed from scratch) to enable readers to understand the core principles and power of Bayesian networks. However, the focus is on ensuring that readers can build practical Bayesian network models, rather than understand in depth the underlying algorithms and theory. Indeed readers are provided with a tool that performs the propagation, so they will be able to build their own models to solve real-world risk assessment problems.

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Preface

The era of 'big data' offers enormous opportunities for societal improvements. There is an expectation – and even excitement – that, by simply applying sophisticated machine learning algorithms to 'big data' sets, we may automatically find solutions to problems that were previously either unsolvable or would incur prohibitive economic costs.

Yet, the clever algorithms needed to process big data cannot (and will never) solve most of the critical risk analysis problems that we face. Big data, even when carefully collected, is typically unstructured and noisy; even the 'biggest data' typically lack crucial, often hidden, information about key causal or explanatory variables that generate or influence the data we observe. For example, the world's leading economists failed to predict the 2008–2010 international financial crisis because they relied on models based on historical statistical data that could not adapt to new circumstances even when those circumstances were foreseeable by contrarian experts. In short, analysts often depend on models that are inadequate representations of reality – good for predicting the past but poor at predicting the future.

These fundamental problems are especially acute where we must assess and manage risk in areas where there is little or no direct historical data to draw upon; where relevant data are difficult to identify or are novel; or causal mechanisms or human intentions remain hidden. Such risks include terrorist attacks, ecological disasters and failures of novel systems and marketplaces. Here, the tendency has been to rely on the intuition of 'experts' for decision-making. However, there is an effective and proven alternative: the *smart data* approach that combines expert judgment (including understanding of underlying causal mechanisms) with relevant data. In particular *Bayesian Networks (BNs)* provide workable models for combining human and artificial sources of intelligence even when big data approaches to risk assessment are not possible.

BNs describe networks of causes and effects, using a graphical framework that provides rigorous quantification of risks and clear communication of results. Quantitative probability assignments accompany the graphical specification of a BN and can be derived from historical data or expert judgment. A BN then serves as a basis for answering probabilistic queries given knowledge about the world. Computations are based on a theorem by the Reverend Thomas Bayes dating back to 1763 and, to date, provides the only rational and consistent way to update a belief in some uncertain event (such as a decline in share price) when we observe new evidence related to that event (such as better than expected earnings). The problem of correctly updating beliefs in the light of new evidence is central to all disciplines that involve any form of reasoning (law, medicine, and engineering as well as finance and indeed AI). Thus, a BN provides a general approach to reasoning, with explainable models of reality, in contrast to big data approaches, where the emphasis is on prediction rather than explanation, and on association rather than causal connection.

BNs are now widely recognized as a powerful technology for handling risk, uncertainty, and decision making. Since 1995, researchers have incorporated BN techniques into software products, which in turn have helped develop decision support systems in many scientific and industrial applications, including: medical diagnostics, operational and financial risk, cybersecurity, safety and quality assessment, sports prediction, the law, forensics, and equipment fault diagnosis.

A major challenge of reasoning causally is that people lacked the methods and tools to do so productively and effectively. Fortunately, there has been a quiet revolution in both areas. Work by Pearl (Turing award winner for AI), has provided the necessary philosophical and practical instruction on how to elicit, articulate and manipulate causal models. Likewise, our work on causal idioms and dynamic discretization has been applied in many application areas to make model building and validation faster, more accurate and ultimately more productive. Also, there are now software products, such as AgenaRisk, containing sophisticated algorithms, that help us to easily design the BN models needed to represent complex problems and present insightful results to decision makers. Compared to previous generations of software these are more powerful and easier to use – so much so that they are becoming as familiar and accessible as spreadsheets became in the 1980s. Indeed, this big leap forward is helping decision makers think both graphically, about relationships, and numerically, about the strength of these relationships, when modelling complex problems, in a way impossible to do previously.

This book aims to help people reason causally about risk and uncertainty. Although it is suitable for undergraduate courses on probability and risk, it is written to be understandable by other professional people generally interested in risk assessment and decision making. Our approach makes no assumptions about previous probability and statistical knowledge. It is driven by real examples that are introduced early on as motivation, with the probability and statistics introduced (from scratch) as and when necessary. The more mathematical topics are separated from the main text by comprehensive use of boxes and appendices. The focus is on applications and practical model building, as we think the only real way to learn about BNs is to build and use them.

Many of the examples in this book are influenced by our academic research but also by our experience in putting the ideas into practice with commercial and government decision and policy makers. Together we have consulted and supplied software to a wide variety of commercial and government organizations, including Milliman LLP, Exxon Mobil, Medtronic, Vocalink, NATO, Sikorsky, World Agroforestry Centre, Virgin Money, TNO, Royal Bank of Canada, Bosch, KPMG, QinetiQ, RGA, GTI, EDF Energy, Boeing and Universities worldwide. We are firm believers in technology transfer and putting useful decision-making theory into the hands of those at the sharp end.

Although purists have argued that only by understanding the algorithms can you understand the limitations and hence build efficient BN models, we overcome this by providing pragmatic advice about model building to ensure models are built efficiently. Our approach means that the main body of the text is free of the intimidating mathematics that has been a major impediment to the more widespread use of BNs.

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