Chapter 5 Exercises

Exercise 1: Mr. Dupont is a professional wine taster. When given a French wine, he will identify it with probability 0.9 correctly as French, and will mistake it for a Californian wine with probability 0.1. When given a Californian wine, he will identify it with probability 0.8 correctly as Californian, and will mistake it for a French wine with probability 0.2. Suppose that Mr. Dupont is given ten unlabelled glasses of wine, three with French and seven with Californian wines. He randomly picks a glass, tries the wine, and solemnly says: "French". What is the probability that the wine he tasted was Californian?

Exercise 2: As accounts manager in your company, you classify 75% of your customers as "good credit" and the rest as "risky credit" depending on their credit rating. Customers in the "risky" category allow their accounts to go overdue 50% of the time on average, whereas those in the "good" category allow their accounts to become overdue only 10% of the time. What percentage of overdue accounts are held by customers in the "risky credit" category?

Exercise 3: A cab was involved in a hit and run accident at night. Two cab companies, the Green and the Blue, operate in the city. You are given the following information:

> 85% of the cabs in the city are Green and 15% are Blue.
>
> A witness identified the cab as Blue. The court tested the reliability of the witness under the same circumstances that existed on the night of the accident and concluded that the witness correctly identified each one of the two colours 80% of the time and failed 20% of the time.

What is the probability that the cab involved in the accident was Blue rather than Green?

Exercise 4: You are on a jury in a murder trial. After a few days of testimony, your probability for the defendant being guilty is 0.8. Then, at the end of the trial, the prosecution presents a new piece of evidence fresh form the lab. The defendant's blood type is found to match that of blood found at the scene of the crime, which could only be the blood of the murderer. The particular blood type occurs in 5% of the population. What should be your revised probability that the defendant is guilty?

Refer to your notes on the Monty Hall games show problem discussed in Chapter 4 and calculate the optimum strategy using Bayes Theorem.

Exercise 5: It is known that there is a single terrorist in a room of 100 people. A lie detection machine, which is 95% accurate, is used to identify the terrorist. What is the chance that the correct person is identified? Make clear any assumptions you use to come up with a solution.

Exercise 6: Based on a previous conversation with a work colleague, you know that he has a daughter at University. You subsequently discover that he has two children. What is the probability the other child is a girl?

Exercise 7: **("The Rule of 5")** Suppose you want to be measure something about a population that has never been measured before, e.g. the total number of minutes a person typically spent in a car yesterday. The population could be your set of friends, your organisation, your town, country, or the whole world. Suppose you can get a random sample of just FIVE people from the population to honestly answer the question. Show that there is a 93.75% probability that the population median for number of minutes spent in a car yesterday lies between the lowest and highest number from your sample of 5. Comment on the ramifications of this result.

Exercise 8: Suppose that a man is charged with a gambling offence, namely that he was using a 'fixed' die in which five of the six sides were 6's. Let H_p be the (prosecution) hypothesis that the die was fixed, and let H_d be the alternative (defence) hypothesis that the die was not fixed (i.e. it was a 'fair' die). A key piece of evidence E presented by the prosecution is the observation that the outcome of two consecutive rolls of the die were two 6s. Calculate the prosecution likelihood, i.e. the probability $P(E|H_p)$ and the defence likelihood, i.e. the probability $P(E|H_d)$, stating any assumptions you are making. Hence, calculate the likelihood ratio of the evidence. What can you conclude about the (posterior) probability of guilt if a) the prior probability $P(H_p) = 0.5$; b) the prior probability $P(H_p) = 0.001$?

Exercise 9: This is similar to the scenario in exercise 8, except in this case the prosecution hypothesis is that the defendant was using a 'fixed' die in which five of the six sides were 6's and the other side was a 5. This time the evidence E is the observation that the outcome of two consecutive rolls of the die were two 5s. Show that the likelihood ratio in this case is 1.

Exercise 10: Use Bayes' theorem to prove that, in general, if H_d is equal to *not* H_p (i.e. H_d and H_p are mutually exclusive and exhaustive) then a likelihood ratio of 1 means that the prior probabilities remain unchanged after observing E (in other words the evidence is truly 'neutral'). Use the same method to show that a likelihood ratio of greater than 1 means that $P(H_p|E) > P(H_p)$ (in other words the evidence truly 'supports' H_p).

Exercise 11: (This highlights an important limitation of the likelihood ratio if the hypotheses are not mutually exhaustive and exclusive). The defendant rolls two dice – a black die which he owns and a red die randomly selected by a member of the public from a batch provided by a reputable dice company. The evidence E against the defendant is that both dice rolls are 6s. However, in this case the prosecution hypothesis H_p concerns only the black die, while the defence hypothesis H_d concerns only the red die:

H_p: "The black die is fixed with all sides being 6s"
H_d: "The red die is fixed with all sides being 6s"

The reason H_d is the defence hypotheses is because it was subsequently discovered that the red die came from a batch in which 50% were faulty in the sense of having all sides sixes. Hence, the prior $P(H_d)=1/2$. Suppose the prior $P(H_p)=1/2$ because it is known that 50% of the defendant's black dice are fixed with all sides being 6s.
Show that the likelihood ratio in this case is 1 but that the evidence is NOT neutral.