## Chapter 13 Exercises

**Exercise 1**: Suppose that a man is charged with murder. Let $H_p$ be the (prosecution) hypothesis that the man is guilty of murder, and let $H_d$ be the alternative (defence) hypothesis that the man is not guilty of murder. A key piece of evidence *E* presented by the prosecution is that a tiny trace of DNA found at the crime scene matches the defendant's DNA profile. It is known that the DNA trace found would match the DNA profile of approximately 1 in every 1000 men. Calculate the likelihood ratio of the evidence *E*, stating clearly all assumptions you are making. What can you conclude about the (posterior) probability of guilt if a) the prior probability $P(H_p) = 0.1$; b) the prior probability $P(H_p) = 0.000001$? Provide some scenarios under which the assumptions you made would be unrealistic.

**Exercise 2**: Suppose a defendant is charged with murder and that:

> $H_p$ is "defendant guilty" and
> $H_d$ is "defendant not at the crime scene"

Then $H_p$ and $H_d$ may ***both*** be true (as would be the case if the defendant paid a hired killer). It is also the case that neither may be true. Suppose the priors for $H_p$ and $H_d$ are both 0.5. Now suppose we get the following evidence *E*

> *E*: Ten minutes before the crime took place the defendant – seen at a different location - was overheard on the phone saying 'go ahead and kill him'.

Assume that both $P(E \mid H_p)$ and $P(E \mid H_d)$ are equal (so the LR is 1). By building a suitable BN show that $P(H_p \mid E) = P(H_d \mid E) = 0.666$. What can you conclude about the LR in this case and the popular notion that a LR of 1 means the evidence is neutral? By extending the BN model to incorporate separate evidence, such as a murder motive, to support $H_p$ show that the probative value of the supposedly 'neutral' evidence *E* can become even more dramatic.

**Exercise 3:** Open the example model "13.9 Vole case". Enter observations in the order shown in Table 13.1 and run the model after each step.

**Exercise 4:** As is Exercise 1 the evidence presented by the prosecution is a 'match' of a trace found at the scene with the defendant's DNA profile (we will call profile of the DNA trace 'X'). However, it is now recognised that there may be errors in the DNA sampling and testing. Hence we have the following distinct hypotheses and evidence:

- **Prosecution hypothesis (*H1*):** "The defendant is the source". The defence hypothesis is simply the negation not *H1*.
- **Evidence *E1***: "The source profile is tested to be of Type X" (note: we can no longer assume the source profile actually is type X)
- **Evidence *E2***: "The defendant profile is tested to be of Type X (note: we can no longer assume the defendant profile actually is type X)

Because of the probability of false positives we cannot assume from the above evidence that either the source or the defendant have type X. Instead these assertions are also unknown hypotheses:

- **Source type hypothesis (*H2*):** "The source profile really is Type X"
- **Defendant type hypothesis (*H3*):** "The defendant profile really is Type X"

Build a BN with 5 associated Boolean nodes to calculate the posterior probability of H1 given the evidence E1 and E2. You can assume that the false positive testing probability is 0.1 and the false negative testing probability is is 0.01. Compare your results with the case where perfect testing accuracy is assumed.